The National IOR Centre of Norway
Post Doc report (Birane Kane), part of

Adding more physics, chemistry and geological realism into the reservoir simulator

Project 2.6.1

Project manager: Ove Sævareid
PhD student: Birane Kane
Project duration: June 2018 – Dec 2021

Final Project Report

Modelling and simulation of non-Newtonian fluids models with application to IOR

Project number and location (UiS, NORCE, IFE): 2.6.1 NORCE
Project duration: June 2018 – Dec 2021
Project manager: Ove Sævareid
PhD students and postdocs: Birane Kane
Other key personnel:

1. Executive summary

This project addresses the modelling and simulation of non-Newtonian fluids with applications to IOR and drilling and well technology. Non-Newtonian fluid flow modelling is acutely important in many industrial and medical application from enhanced oil recovery to blood flow in arteries and polymer processing. The behaviour of those non-Newtonian flows is complex and requires a careful treatment of the physical processes involved as well as accurate and efficient numerical techniques.

Within this project, we first provide a flexible and state-of-the-art framework based on a classical scientific computing approach. We also apply the new concept of Physics-informed Machine Learning to non-Newtonian flow simulation.

2. Introduction and background

There are many models aiming at describing non-Newtonian flow, however, we focus on two of viscoelastic type, namely the Olroyd-B type [1] and the FENEP (Finitely Extensible Nonlinear Elastic) model [2]. In contrast to Newtonian (resp. quasi-Newtonian) fluids where the stress tensor is a linear (resp. nonlinear) functional of the velocity and pressure field, the viscoelastic flows require to consider another constitutive equation where the stress tensor is another unknown. Simulation of such problem class becomes quite challenging when the amount of fluid elasticity is highly increased. Thus, requiring stabilization techniques to avoid failure of the numerical simulation or spurious oscillations. This phenomenon is known as the High Weissenberg Number problem (HWNP) [3]. Another challenge comes from the hyperbolic constitutive equation when the advection term becomes dominant. In the context of Finite element methods, various strategies have been considered. We can mention the SUPG (Streamline Upwind Petrov Galerkin) method of Marchal and Crochet [4] and its non-consistent and only first order accurate counterpart the SU (Streamline Upwind) method. Discontinuous Galerkin methods are also a quite attractive approach. They have been considered by Fortin et Fortin [8], Baijens[9] and Yurun [10], where the pre-eminence of DG methods over the SUPG was shown. In [Baijens [9], a finite element approach fulfilling the Ladyzenskaya Babuska Brezzi conditions is considered for the mass and momentum conservation equations while DG is used for the hyperbolic constitutive equation. Unfortunately, all these traditional numerical methods for solving PDEs, such as finite difference, finite elements and Galerkin methods have mostly failed to address the shortcomings fully and efficiently.

Recently, Raissi et al. [11], demonstrated that it is possible to combine Machine Learning approaches with more traditional physics approaches. These so-called Physics-informed Machine Learning approaches are designed to obtain solutions of general nonlinear PDEs, and they may be a promising alternative to traditional numerical methods for solving PDEs, such as finite difference and finite elements methods. The core idea of PINN is to explicitly embed the physical laws (e.g., the governing partial differential equations, initial/boundary conditions, etc.) into a deep neural network, constraining the network's trainable parameters within a feasible solution space.

3. Results

The main result of this project resides in the establishment of flexible simulation frameworks. Our initial focus consisted in providing a state-of-the-art Finite Element/Discontinuous Galerkin discretization module within the DUNE toolbox [12]. Further, we focused on the study of innovative approaches based on the application of deep learning techniques to viscoelastic PDEs.

3.1 Study and implementation with high order discretization methods

The first part comprised in implementing a splitting scheme that is used to decouple pressure and velocity followed by solving the constitutive equation for the stress. We used the Taylor-Hood finite element for the Velocity-Pressure System and a DG discretization for the constitutive equation. The implementation is based on the new Python frontend of the Dune-Fem [13,14] to the open-source framework DUNE. We first considered the case of quasi-Newtonian models before extending it to viscoelastic models such as Oldroyd-B and FENEP. We also provided an effective framework for hp-adaptive discretization of such models. Different scheme splitting and projection strategies are also available within the framework. The modules are available on Gitlab: https://gitlab.dune-project.org/birane.kane/dune-visco-fempy

3.2 Study and implementation with Physics-informed Machine Learning

In the second part of the project, we introduced a new framework where we explicitly embed physical laws aiming at describing viscoelastic fluid flow (e.g., Oldroyd/FENEP equations) to constrain neural networks for training a reliable model. The effectiveness of the proposed framework is demonstrated through some benchmark tests. To our knowledge, this is the first time the concept of deep learning is applied to viscoelastic fluid flow modelling. The implementation of our model is based on a suitable open-source numerical modelling platform using the TensorFlow library.

4. Conclusion(s)

The achievement of this project is twofold. In addition to the initial goal of providing a flexible and state-of-the-art framework based on a classical scientific computing approach; we applied the new concept of physics-informed Machine Learning to viscoelastic flow simulation. This pioneering approach is opening new horizons where deep learning methods will be used to improve existing PDE solvers with data from the experimental sites and PDEs be used to form a backbone prior for deep learning methods.

5. Future work/plans

We are currently completing a paper on Physics-informed Machine Learning application to viscoelastic flows. This paper focuses on the effectiveness of deep learning techniques with regards to forward and inverse problems in complex flows. Oldroy-B and FENEP models are studied and evaluated through some benchmark tests. We hope to submit it by the end of this year.

6. Dissemination of results

All the implementation in this project is released open-sourced: https://gitlab.dune-project.org/birane.kane/dune-visco-fempy

7. References

[1] Oldroyd, James G. "On the formulation of rheological equations of state." *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* 200.1063 (1950): 523-541.

[2] Purnode, B., and M. J. Crochet. "Polymer solution characterization with the FENE-P model." *Journal of non-newtonian fluid mechanics* 77.1-2 (1998): 1-20.

[3] Fattal, Raanan, and Raz Kupferman. "Time-dependent simulation of viscoelastic flows at high Weissenberg number using the log-conformation representation." *Journal of Non-Newtonian Fluid Mechanics* 126.1 (2005): 23-37.

[4] Crochet, M. J., and Vincent Legat. "The consistent streamline-upwind/Petrov-Galerkin method for viscoelastic flow revisited." *Journal of non-newtonian fluid mechanics* 42.3 (1992): 283-299.

[5] Franca, Leopoldo P., and Thomas JR Hughes. "Two classes of mixed finite element methods." *Computer Methods in Applied Mechanics and Engineering* 69.1 (1988): 89-129.

[6] Behr, M., et al. "GLS-type finite element methods for viscoelastic fluid flow simulation." *Proc of the Third MIT Conference on Computational Fluid and Solid Mechanics, Massachusetts Institute of Technology, Cambridge, USA*. 2005.

[7] Castillo, E., and Ramon Codina. "Finite element approximation of the viscoelastic flow problem: A non-residual based stabilized formulation." *Computers & Fluids* 142 (2017): 72-78.

[8] Fortin, Michel, and André Fortin. "A new approach for the FEM simulation of viscoelastic flows." *Journal of non-newtonian fluid mechanics* 32.3 (1989): 295-310.

[9] Baaijens, Frank PT. "Application of low-order discontinuous Galerkin methods to the analysis of viscoelastic flows." *Journal of Non-Newtonian Fluid Mechanics* 52.1 (1994): 37-57.

[10] Yurun, Fan. "A comparative study of the discontinuous Galerkin and continuous SUPG finite element methods for computation of viscoelastic flows." *Computer methods in applied mechanics and engineering* 141.1-2 (1997): 47-65.

[11] Raissi, Maziar, Paris Perdikaris, and George E. Karniadakis. "Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations." *Journal of Computational Physics* 378 (2019): 686-707.

[12] Bastian, Peter, et al. "The Dune framework: Basic concepts and recent developments." *Computers & Mathematics with Applications* 81 (2021): 75-112.

[13] Dedner, Andreas, et al. "A generic interface for parallel and adaptive discretization schemes: abstraction principles and the DUNE-FEM module." *Computing* 90.3 (2010): 165-196.

[14] Dedner, Andreas, et al. "Python framework for hp-adaptive discontinuous Galerkin methods for two-phase flow in porous media." *Applied Mathematical Modelling* 67 (2019): 179-200.